Scaling Functions of Polymer-Induced Turbulent Drag Reduction Focusing on the Polymer–Solvent Interaction

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ABSTRACT: Based on turbulent drag reduction characteristics of polystyrene and polyisobutylene in a pipe flow and a rotating-disk flow, respectively, a relationship between polymer concentration and drag reduction at a given Reynolds number was considered. The universal drag reduction equation of a three-parameter relationship between drag reduction and polymer concentration was modified using intrinsic concentration and intrinsic viscosity, and it was then found to be the most useful formula for correlating DR data, especially for polymer–solvent interactions in a turbulent flow. © 2003 Wiley Periodicals, Inc. J Appl Polym Sci 88: 1836–1839, 2003

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The addition of small amounts of high molecular weight polymers (typically on the order of parts per million) to a turbulent flow can yield a drastic increase in the mass flow at a given pressure gradient. The drag reduction (DR) is defined as the amount of the reduction in the skin-frictional drag. Polymer-induced DR has been the subject of numerous theoretical and experimental studies, but despite these efforts, the details of the underlying physical and chemical mechanisms of DR have not fully been investigated. DR is known to be strongly influenced by the molecular properties of the dissolved polymer. We introduce a universal scaling function that describes DR correlation by focusing on the effects of the polymer molecular parameters and solvent quality. DR efficiency can be described through a consideration of the critical concentration above which no DR will occur.

The purpose of this work is to examine the relationship between the polymer concentration (C) and DR at a given Reynolds number. We examine the effects of the polymer molecular parameters and solvent quality on DR, this being useful for unifying the DR correlation.

The concentration-dependent DR profile was historically examined by Virk et al.[1] and Little and coworkers.[2,3] Virk et al. developed the first universal DR relationship through a homologous series of poly(ethylene oxide) (PEO) in water as a function of the concentration, molecular weight, and flow rate in the internal flow. Later, Little and coworkers[2,3] simplified and rearranged this universal relationship, correlating DR to the external flow situation. Their universal relationship provides a means of correlating the DR performances of various polymers.[3] Using C, the maximum DR (DR_{max}), which is used to normalize DR (DR/DR_{max} is defined as the normalized DR), and the intrinsic concentration ($[C]$), which is defined as $[C] = DR_{max}/\lim_{C \to 0}(DR/C)$ these investigators established the following two-parameter empirical equation to describe the DR performance:

$$\frac{DR}{DR_{max}} = \frac{C}{[C] + C} \tag{1}$$

To fit the DR data, eq. (1) is generally rearranged into the following form for ranges up to the optimum concentration:

$$\frac{C}{DR} = \frac{[C]}{DR_{max}} + \frac{C}{DR_{max}} \tag{2}$$

Equation (2) suggests that there exists a linear relationship between C/DR and C. This is qualitatively valid for most drag reducers for both pipe (internal) and rotating-disk (external) flows.[3–6]

Figure 1 demonstrates the linear relationship between C/DR and C for linear polystyrene (PS) samples with three different molecular weights with the data of Gramain and Borreill.[6] The empirical parameters
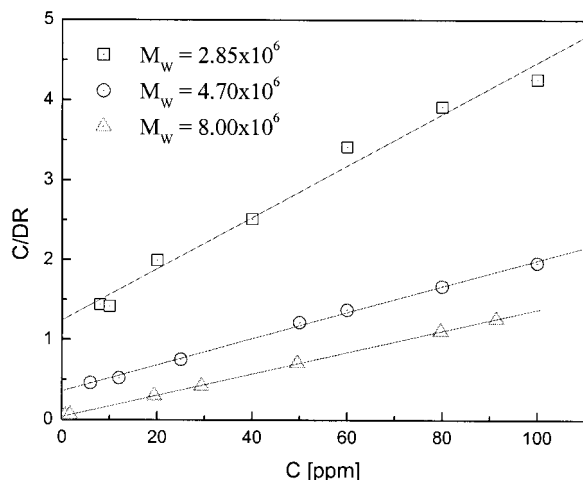
Figure 1 C/DR versus C for PS samples with three different molecular weights in toluene at a Reynolds number of 200,000 in a pipe flow with a 2-mm inner diameter.[6]

DR_{max} and $[C]$, descriptive for each polymer–solvent interaction and for the flow properties, have been obtained from eq. (2). Generally, the larger DR_{max} is and the smaller $[C]$ is, the more effective the drag reducer is. From Figure 1, we obtained values of DR_{max} for PS at various weight-average molecular weights (M_w's): 0.31 for $M_w = 2.85 \times 10^6$, 0.61 for $M_w = 4.70 \times 10^6$, and 0.75 for $M_w = 8.00 \times 10^6$. These values can be used to correlate the molecular parameters and turbulent strength if there is no degradation. Zakin and Hunston[7] reported the effect of a diluent, finding that the addition of a low molecular weight polymer to a polymer mixture had little effect on DR_{max} but greatly influenced $[C]$.

Choi and Jhon[4] improved the DR data fit with a three-parameter relationship between DR and C (i.e., they introduced on additional parameter, $K \neq 1$, quantifying the polymer–solvent interaction) by casting DR into a Padé form:

$$\frac{C}{DR} = \frac{K[C]}{DR_{max}} + \frac{C}{DR_{max}} \quad (3)$$

Equation (3) is an improvement over the fit obtained by previous investigators. For a PEO–water system, $K = 1$ fits the data reasonably well and agrees with the previous results.[1–3] However, $K = 0.4$ gives the best fit for polyisobutylene (PIB) samples of different molecular weights in toluene for a pipe flow.[8] Our extensive experimental efforts also confirmed that PIB in different solvents exhibits different K values.[9] We found that the parameter K is independent of the molecular weight and flow geometry but is strongly dependent on the polymer–solvent interaction.[10] Therefore, eq. (3) was used in the past to examine polymer–solvent interactions in a turbulent flow.

Other investigators correlated the DR performance through the intrinsic viscosity ($[\eta]$) rather than $[C]$. McCormick and coworkers[5,11] normalized DR with the polymer volume fraction ($[\eta]C$) for a series of water-soluble synthetic polymers to examine the effects of polymer solution properties on the DR performance. In their extensive efforts, they obtained a universal curve by introducing a shift factor to collapse the vast amount of their DR data. It is remarkable that their scaling theory based on $[\eta]$, which is the intrinsic material property at a low Reynolds number, gives a universal correlation for a broad range of experimental conditions.

$[\eta]$ accounts for the polymer molecular weight and polymer–solvent interaction. The most effective drag reducers in their studies were those having the largest values of $DR/[\eta]C$ and/or the lowest values of $[\eta]C$ (the volume fraction). The Mark–Houwink relationship yields the dependence of $[\eta]$ on M_w, that is, $[\eta] = \alpha M_w^\beta$ (where α and β are the flow-independent properties), and it is determined from the given polymer, solvent, and temperature.

$[C]$ is related to the polymer molecular weight as the form analogous to the Mark–Houwink relationship,[3,8] which yields a power-law dependence similar to that of $[\eta]$ on the molecular weight. In our series of experimental efforts, we found that $1/K[C] = AM_V^B$. Here, unlike the Mark–Houwink parameters, A and B are parameters descriptive of the flow properties but weakly dependent on solution properties.

With this relationship, eq. (3) can be rearranged as follows:

$$\frac{1}{1 - DR/DR_{max}} - 1 = \frac{DR/DR_{max}}{1 - DR/DR_{max}}$$

$$= \frac{C}{K[C]} = AM_v^B C \quad (4)$$

Note that most of the DR-versus-C data in the literature use information on $[C]$ rather than on $[\eta]$ because of the effect of the Reynolds numbers. Also note that the correlation with $[\eta]$ appears to be valid only at a very low Reynolds number.

In this article, we have established a unified description between these two different approaches by simply recognizing the relationship between $[C]$ and $[\eta]$.

Water-soluble polymer families, including PEO and poly(acrylic acid), exhibit a linear relationship between $[C]$ and $[\eta]$ in a log plot.[3] $1/K[C]$ is related to $[\eta]$ as follows:

$$\ln(1/K[C]) = \ln A + B\ln M_v = \ln A + B\left(\frac{\ln[\eta] - \ln \alpha}{\beta}\right)$$

$$(5)$$

Substituting eq. (5) into eq. (4), we obtain the relationship between DR and C via $[\eta]$:

$$\ln\left(\frac{\mathrm{DR}/\mathrm{DR_{max}}}{1 - \mathrm{DR}/\mathrm{DR_{max}}}\right) = \ln A' + B' \ln[\eta] + \ln C \quad (6a)$$

$$\ln A' = \ln A - \ln \alpha/\beta \text{ and } B' = B/\beta \quad (6b)$$

If $A' = B' = 1$, we recover the results given in ref. 11. Figure 2 illustrates how eqs. (6a) and (6b) produce a universal correlation from the existing DR data. We can obtain A' and B' from the intercept in Figure 2(a) and B' from the slope of eq. 2(b), and then we can rearrange eq. (6a) to obtain the relationship of C/DR versus C, which is analogous to eq. (3):
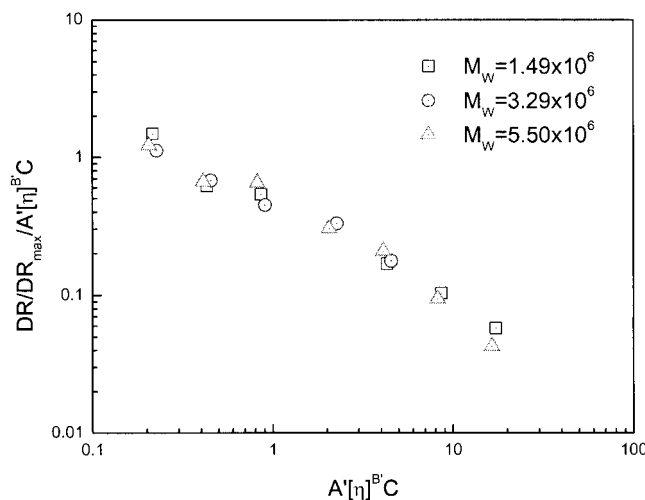
$$\frac{\mathrm{DR}}{\mathrm{DR_{max}}} = \frac{A'[\eta]^{B'}C}{1 + A'[\eta]^{B'}C} \quad (7)$$
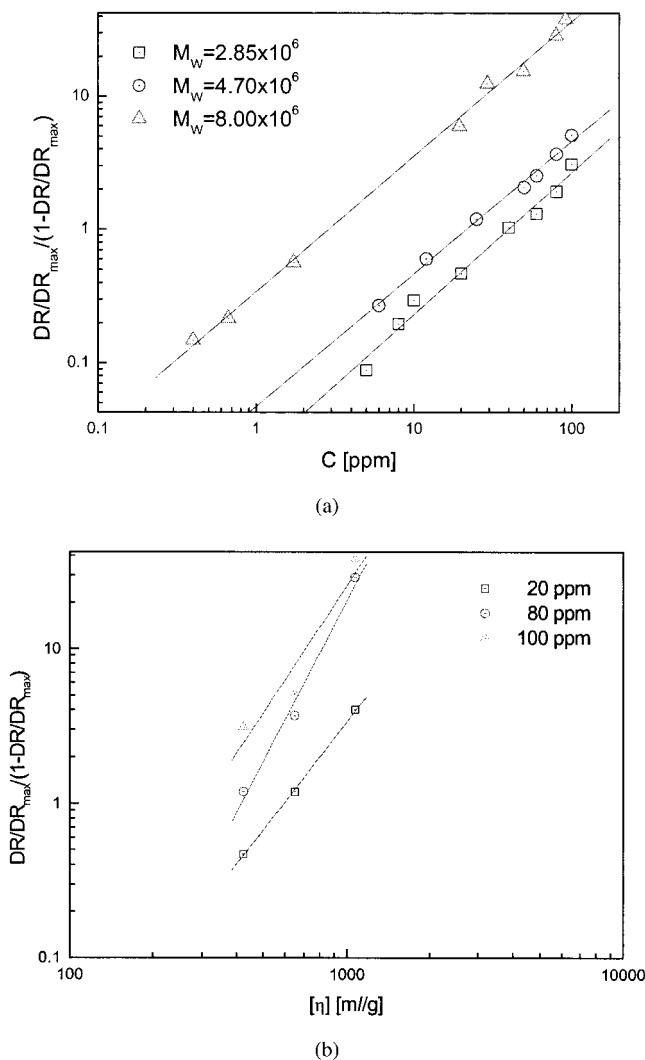
Figure 2 Plots (a) for PS samples with three different molecular weights in toluene at a Reynolds number of 200,000 in a pipe flow with a 2-mm inner diameter as a function of C [A' was obtained from eq. (6a)] and (b) for PS samples with three different values of C in toluene at a Reynolds number of 200,000 in a pipe flow with a 2-mm inner diameter as a function of [η] [B' was obtained from eq. (6a)].
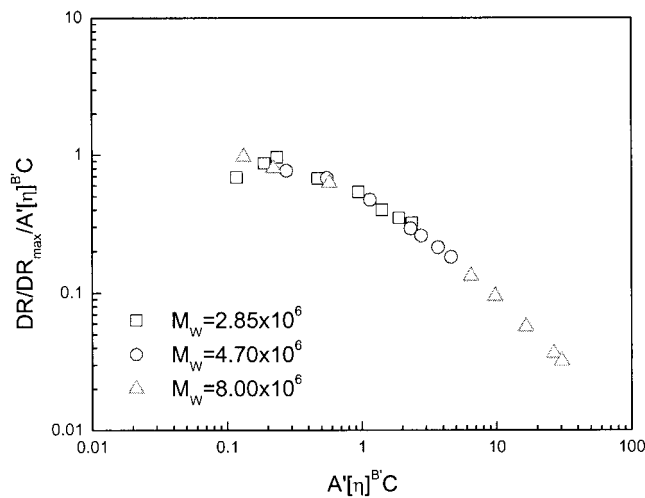
Figure 3 Normalized DR per $A'[\eta]^{B'}C$ versus $A'[\eta]^{B'}C$ showing (a) the universality for PS samples with three different molecular weights in toluene at a Reynolds number of 200,000 in a pipe flow with a 2-mm inner diameter and (b) the universality for PIB samples with three different molecular weights in cyclohexane in a rotating-disk flow.[10]

or

$$\frac{C}{\mathrm{DR}} = \frac{1}{A'[\eta]^{B'}\,\mathrm{DR_{max}}} + \frac{C}{\mathrm{DR_{max}}} \quad (8)$$

Here, $A'[\eta]^{B'}C$ is identified as the modified volume fraction (i.e., it reduces to the result of McCormick et al.[5] for $A' = B' = 1$). Therefore, one can use either eq. (3) or eq. (8) to correlate C/DR versus C and to provide molecular design criteria for selecting the optimal drag reducer. The slope obtained from either equation gives $\mathrm{DR_{max}}$, whereas the intercept is related to the polymer–solvent interaction in a turbulent flow via [C] or [η].

Although this derivation is straightforward, it may be the most useful formula for correlating DR data,

especially for polymer–solvent interactions in a turbulent flow. Figure 3 illustrates the DR efficiency with the PS concentration via eq. (8). Our preliminary study indicates a universality with a broad range of material characteristics and experimental conditions. An extensive data analysis with eqs. (3) and (8) will be published elsewhere.

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